

# Theoretical Computer Science

$$\lambda \mapsto \int e^{i\lambda x} f(x) dx$$

79 November 2029

# What does it mean to compute?

- ▶ Computation means following a set of instructions.
- ▶ Originally, theoretical computer science was concerned only with (Turing / von Neumann) computers.
- ▶ The field has matured, and is beginning to export its knowledge. (Daskalakis, Arora, Childs (Quantum).)
- ▶ (Most of the field is still doing what it did ten years ago: approximation algorithms, complexity, optimization.. but I'll focus on recent fancy stuff.)

# The world according to ..

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**LATEST:** Sarah Palin sneezes, snot sold for £2,000,000



## G20 summit stretches to seventieth day

Politicians act like five-year-olds in the face of financial strains and China owning the west in every conceivable manner.

## Financial crisis predicted

Bankers still treat constituents like endless coffers and wonder how many yachts they can squeeze into their private harbors.

## Lady Gaga concert cancelled

Collapses prior to concert, citing fatigue and ill-adjustment to new tour schedule.

# The world according to .. a CS theorist.

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**LATEST:** International holiday commemorating Grigori Perelman



## G20 summit stretches to seventieth day

Desired consensus is a Nash equilibrium, and therefore PPAD-hard; deliberations will last forever. (Daskalakis et al.)

## Financial crisis predicted

Detecting unfairness in a set of CDOs is equivalent to the dense subgraph problem and therefore NP-hard. (Arora et al.)

## Lady Gaga concert cancelled

Finding optimal chord progression is reducible to knapsack and therefore NP-hard.

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## Algorithmic game theory.

- ▶ To prove the existence of Nash Equilibria, the Brouwer fixed point theorem was used. Was this necessary? (in a sense of computational equivalence.)
- ▶ It turns out, YES: both are complete for the same complexity class (PPAD, a subset of FNP).
- ▶ Constantinos Daskalakis won the 2008 ACM Dissertation award for “The Complexity of Nash Equilibria”.
- ▶ Papadimitriou: “If my laptop can’t compute it, neither can the market”.
- ▶ Lots of research into better equilibria concepts.
- ▶ Lots of internet companies hire game theorists, e.g. for their ad auctions.

## UGC-hardness, the new holy grail.

- ▶ Theorists care about SDPs: 0.878-factor approximation to max-cut via SDP.
- ▶ Unique Games Conjecture (roughly): determining whether a (an exotic) graph property is fractionally satisfiable is NP-hard. (Details if time..)
- ▶ 0.878 looks ridiculous.. under  $UGC + (P \neq NP)$ , it is optimal.
- ▶ UGC says that for a wide variety of problems, the SDP is the optimal algorithm, and the integrality gap is thus the approximation ratio.

## Smoothed analysis.

- ▶ To solve a linear program, can use an interior point method ( $O(s^{3.5} \log(1/\epsilon))$ ) or the simplex method ( $2^{\Omega(n)}$ )... but simplex is fast in practice!!
- ▶ The point: for some algorithms and their typical settings, these bad instances are rare.
- ▶ Smoothed analysis: convolve optimization space with a gaussian, read off the “smoothed” worst case.
- ▶ Key results: simplex and  $k$ -means are smoothed-poly.
- ▶ Won Teng+Spielman the 2008 Gödel prize!
- ▶ Interesting aside: Vempala has a new algorithm (ICS'10) which solves strong linear programming. No one knows the running time yet, but it seems to defeat typical bad cases.

## Graph partitioning.

- ▶ Classical problem: max-flow/min-cut. (Still open! Lower bound  $\Omega(m)$ , upper bound Spielman/Daich  $\tilde{O}(m^{1.5} \log(w_{\max}/\epsilon))$ .)
- ▶ This cut may be unbalanced: divide by smaller size or inner connectivity! (“Sparsest Cut”, ”Normalized Cuts”.)
- ▶ Spectral methods: look at eigenspace of matrix representation. (first alg for normalized cuts.)
- ▶ OR: do some matrix math, write down approximating SDP. (Can solve it with boosting!)

## Miscellaneous gossip.

- ▶ STOC/FOCS are the top conferences, but many felt it became a technical proving ground for grad students; ICS was started (this year) to give a place to show new ideas.
- ▶ The Arora result alluded to in the joke was presented at ICS, and also at UCSD.
- ▶ Princeton/IAS/Rutgers/Courant received a huge grant for pure complexity theory (with no concrete stated goals!).
- ▶ Privacy preserving operations.

## Top theory schools.

- ▶ MIT. Optimization (Goemans), Complexity (Sudan), Crypto (Goldwasser), Quantum (Aaronson). (indyk rivest sipser daskalakis..)
- ▶ Berkeley. Quantum (U. Vazirani), Games (Papadimitriou), Crypto/Complexity (Trevisan).
- ▶ Princeton/IAS. Everything (Arora), Complexity (Widgerson).
- ▶ CMU. Boolean functions (O'Donnell), Approximation Algorithms (Gupta/other dude), being smarter than everyone (A. Blum).
- ▶ Other strong places: UWashington (James Lee), Georgia Tech (Prasad Raghavendra, Santosh Vempala), Microsoft Research Cambridge (Mark Braverman, Adam Kalai, Madhu Sudan).

## Theory @ UCSD.

- ▶ ♡♡ Russell Impagliazzo ♡♡: is randomness inherently more powerful?
- ▶ Sanjoy Dasgupta: wrote algorithms textbook lots of schools use.
- ▶ Yoav Freund: Boosting  $\rightarrow$  multiplicative weights.
- ▶ Mihir Bellare: fundamental results in crypto (theory people think he is awesome).
- ▶ Daniele Micciancio: applying hard problems to build crypto systems (paper with Panos on shortest vector appeared in SODA this year).
- ▶ Mohan Paturi: recent focus on social networks.
- ▶ Fan Chung Graham: spectral graph theory.



# The Unique Games Conjecture

A **unique game** is a graph  $G = (V, E)$ , a set of admissible colors  $C$ , and a set of permutations  $f_e$  constraining  $c(v_1) = f_e(c(v_2))$  where  $e : v_1 \rightarrow v_2$  and  $c(x)$  is the color of vertex  $x$ .

To find an exact coloring takes time  $O(C(m+n))$ ; fix a vertex, and for each color it can take, propagate colors to other vertices in the graph. But seeing if an  $\epsilon$  fraction of the constraints can be satisfied is a pain. In fact..

The **Unique Games Conjecture** is that for every small constant  $\epsilon > 0$ , there exists a  $C$  such that for any unique game, it is  $NP$ -hard to tell if at least  $1 - \epsilon$  of the constraints can be satisfied, or if every color satisfies less than  $\epsilon$ .